Edge-state Fabry-Perot interferometer as a high-sensitivity charge detector

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We present a scheme for high-sensitivity and high-efficiency charge detection in the integer quantum-Hall regime using two point contacts in series. The setup is an electronic analog of an optical Fabry-Perot interferometer. We show that for small transmission through the point contacts the sensitivity of the interferometer is very high due to multiple reflections at the point contacts. The sensitivity can be further enhanced twice by using electrons in spin-entangled state. We show that for point contacts having different reflection probabilities, the detector can be tuned for the quantum-limited measurement.

Measurement of the charge state of a mesoscopic system has generated a lot of interest in recent years,[1–3] mainly due to the applications of charge qubits in solid-state realization of quantum information processing.[4] Mesoscopic devices such as quantum point contact (QPC) (Ref. [5]) and single-electron transistor (SET) (Ref. [6]) have been widely used as the charge detectors. These detectors do not perform instantaneous measurement, but the measurement is performed as a sequence of continuous weak measurements.[7] The merits of these detectors can be understood from the two points of view: (1) efficiency and (2) sensitivity. The former is related to the back-action noise produced by the detector and the latter is related to the precision. The quantum-mechanical complementarity establishes a tradeoff between acquisition of information about the state of the system and the back-action dephasing. A detector is called 100% efficient (quantum limited) if the dephasing occurred in the measured system is only due to the acquisition of information by the detector. On the other hand, performing more sensitive measurements have often led to reveal new physics.[8] A high-sensitivity charge detector working in the quantum limit can have wider applications in quantum metrology.[9] The improvements in measurements can be accomplished either through new designs of measurement devices or by developing methods that rely on properties such as correlations and entanglement.[10–12]

In this Brief Report, we present a model of a high-sensitivity and high-efficiency charge detector in the integer quantum-Hall regime. Our model is an electronic analog of Fabry-Perot interferometer.[13] For fractional quantum-Hall states, a similar arrangement has been proposed for measuring fractional charge and non-Abelian statistics.[14] We show that the charge sensitivity of our model is higher than a two-path interferometer due to multiple reflections of electrons at QPCs. We also report the possibility of tuning the detector for quantum-limited measurement for $R_a < R_b$, where $R_a(R_b)$ is reflection probability of the first (second) quantum point contact QPC_a (QPC_b) (cf. Fig. 1). We note that two-path interferometer, with edge channel (Mach-Zehnder interferometer), has been realized.[15] The conditions of quantum-limited detection of charge using Mach-Zehnder interferometer have also been discussed.[16]

In Fig. 1, we show a schematic setup, constructed using electrical gates on a Hall bar, for measurement of a charge qubit. Our detector consists of two QPCs, QPC_a and QPC_b, arranged in series. The input electrons are injected from the source terminals α and γ. The outgoing electrons are collected at the drain terminals β and δ. In the quantum-Hall regime, QPCs act as the beam splitters for the incoming electrons. The point contact QPC_a splits the incoming edge-state current from source α into two parts with one reflected back to the drain β and the other transmitted to the second point contact QPC_b. The edge-state current on reaching at QPC_b is further split into two parts, one transmitted to the drain δ and other part reached at QPC_a, where it is again partially transmitted to drain β and partially reflected back to QPC_b and so on. Thus our detector is analogous to optical Fabry-Perot interferometry. A charge qubit is capacitively attached to the lower arm of the interferometer between the two QPCs. The qubit, having two charge states $|0\rangle$ and $|1\rangle$, could be a double quantum dot or a two-path interferometer. There is no electron transfer from the qubit to the interferometer. Due to Coulomb interaction the charge on the qubit deflects edge state in the lower arm without changing transmission through QPCs, which modifies the phase of the edge-state current via the Aharonov-Bohm effect.

The information of the measured state of the qubit is reflected in the electrons collected at drain reservoirs. We follow scattering matrix analysis for input-output probability amplitudes. The scattering matrix in terms of Fermi operators at mth terminal c_m, $m = \alpha, \beta, \gamma, \delta$ is written as follows:

$$\begin{pmatrix} c_\beta \\ c_\delta \end{pmatrix} = \begin{bmatrix} \bar{r}_i & \bar{t}'_i \\ \bar{t}_i & \bar{r}'_i \end{bmatrix} \begin{pmatrix} c_\alpha \\ c_\gamma \end{pmatrix}, \tag{1}$$

$$\bar{r}_i = r_a + \frac{t_a t'_a r_b e^{i(\phi + \theta_i)}}{1 - r'_a r_b e^{i(\phi + \theta_i)}}, \quad \bar{t}_i = \frac{t_a t_b e^{i\theta_i}}{1 - r'_a r_b e^{i(\phi + \theta_i)}}, \tag{2}$$
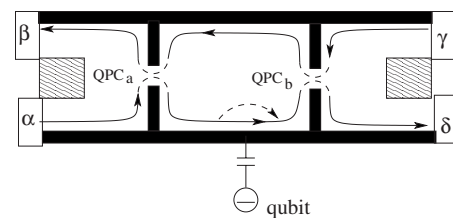

FIG. 1. Schematic arrangement for measurement of a charge qubit. Two spatially separated point contacts form the Fabry-Perot interferometer. The qubit is capacitively attached in one arm of the interferometer.

$$\bar{r}'_i = r'_b + \frac{t_b t'_b r'_a e^{i(\phi+\theta_i)}}{1 - r'_a r_b e^{i(\phi+\theta_i)}}, \quad \bar{t}'_i = \frac{t'_a t'_b e^{i\phi}}{1 - r'_a r_b e^{i(\phi+\theta_i)}}, \quad (3)$$

where ϕ is the Aharonov-Bohm phase acquired by the electron along one complete loop between QPCs and θ_i is the phase produced by the qubit. The phase θ_i has two values corresponding to different charge states of the qubit $|i\rangle$, $i = 0, 1$. Typical value of the phase difference $\Delta\theta = \theta_1 - \theta_0$ generated by the Coulomb interaction is about $\Delta\theta = 0.03$.[3] Effectively, charge state of the qubit modifies the amplitude as well as the phase of the transmission through the detector. All other phases in scattering are included in the transmission amplitudes $t_n(t'_n)$ from the left (right) and the reflection amplitudes $r_n(r'_n)$ on the left (right) for QPC$_n$, where $n = a, b$.

First, we consider that electrons are injected only from the source terminal α and collected at the drain terminal δ. The transmission probability $\bar{T}_i(=|\bar{t}_i|^2)$ of the interferometer is given by

$$\bar{T}_i(\Phi_i) = \frac{T_a T_b}{1 + R_a R_b - 2\sqrt{R_a R_b} \cos\Phi_i}, \quad (4)$$

where $\Phi_i = \theta_i + \phi + \arg(r'_a r_b)$ and $T_n = |t_n|^2 = 1 - R_n$. Dependence of the transmission probability on the variation in phase Φ_i makes it possible to detect the charge state of the qubit. The transmission probability has sharp Lorentzian-type resonances when Φ_i are multiples of 2π. The half width at half maximum of the resonance is $\Gamma_w \approx (1 - \sqrt{R_a R_b})/(R_a R_b)^{1/4}$. The resonances are narrower for larger values of R_a and R_b, which provides larger change in current for small variations in phase Φ_i. Because of quantum complementarity, the phase sensitivity (precision) of the interferometer is determined by the phase fluctuations due to intrinsic shot noise. In the linear regime, the average source-drain current is $\langle I_i \rangle = (e^2 V/h)\bar{T}_i$ and the shot noise is given by $S_i = (2e^3 V/h)\bar{T}_i(1-\bar{T}_i)$, where V is source-drain voltage. For time interval τ, the average number of transmitted electrons is $\langle N_i \rangle = \langle I_i \rangle \tau/e$ and the uncertainty in number of electron is $\langle(\Delta N_i)^2\rangle = S_i\tau/(2e^2)$. Therefore, the rms phase fluctuation[11] for the interferometer is given by

$$\Delta\Phi_i \equiv \frac{\sqrt{\langle(\Delta N_i)^2\rangle}}{|\partial\langle N_i\rangle/\partial\Phi_i|} = \sqrt{\frac{h}{eV\tau}} \frac{\sqrt{\bar{T}_i(1-\bar{T}_i)}}{|\partial\bar{T}_i/\partial\Phi_i|}. \quad (5)$$

From Eqs. (4) and (5) one can calculate the sensitivity of Fabry-Perot interferometer. We compare the sensitivity of Fabry-Perot interferometer with a two-path (Mach-Zehnder) interferometer for which transmission probability is cosine function of the form $\bar{T}_i(\Phi_i) = R_a R_b + T_a T_b + 2\sqrt{R_a R_b T_a T_b} \cos\Phi_i$.[15] Near the resonance, for $R_a \approx R_b$, the ratio of $\Delta\Phi_i$ for Fabry-Perot interferometer to Mach-Zehnder interferometer is approximately $T_a^{3/2}$. Clearly, Fabry-Perot interferometer can be used as a very high-precision charge detector for small transmission probabilities T_a and T_b.

In real devices, this high precision would be limited by the finite source-drain bias voltage because the phase Φ_i acquires an additional energy-dependent fluctuating part.[17]
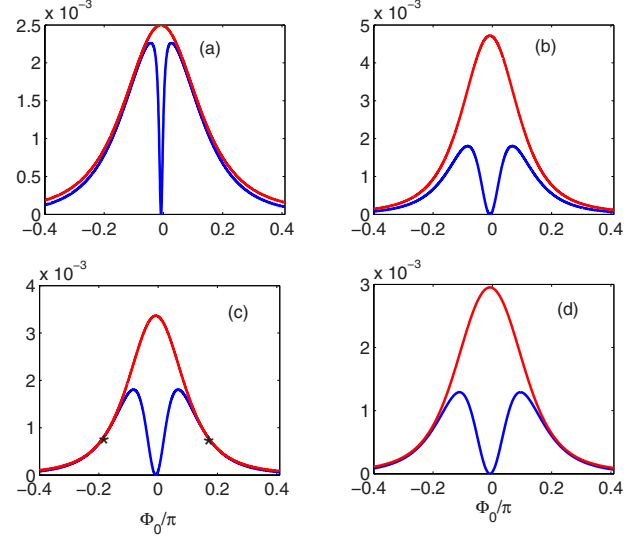

FIG. 2. (Color online) The renormalized measurement rate Γ_m/Γ_0 (lower lines) and dephasing rate Γ_d/Γ_0 (upper lines) for $\Gamma_0 = eV/h$ and $\Delta\theta = 0.05$, (a) for symmetric interferometer ($R_a = R_b = 0.5$), (b) for $R_a > R_b$ i ($R_a = 0.7$, $R_b = 0.5$), and (c) for $R_a < R_b$ ($R_a = 0.5$, $R_b = 0.7$). The interferometer operates in quantum limit for $\Phi_0 = \pm\cos^{-1}\sqrt{R_a/R_b}$, shown as asterisks. (d) Same as (c) for small finite bias ($eV/E_c = 0.5$). Note that $\Gamma_w \approx 0.53$ for $R_a = 0.5$ and $R_b = 0.7$.

Considering drift velocity v_d as constant along the edges, we can write energy dependence of phase as $\Phi_i(\epsilon) = \Phi_i(E_F) + \epsilon/E_c$, $E_c = \hbar v_d/L$, where L is the length of one complete loop between the QPCs, E_F is Fermi energy and ϵ is small energy difference for electrons from Fermi level. The averaging of the energy-dependent fluctuations gives average transmission probability and average shot noise, respectively, as $\langle \bar{T}_i \rangle = (eV)^{-1}\int_{-eV/2}^{eV/2}\bar{T}_i[\Phi_i(\epsilon)]d\epsilon$ and $\langle S_i \rangle = 2e^3/h\int_{-eV/2}^{eV/2}\bar{T}_i[\Phi_i(\epsilon)]\{1-\bar{T}_i[\Phi_i(\epsilon)]\}d\epsilon$. At small bias ($eV/E_c < \Gamma_w$), we find that $\Delta\Phi_i$ is changed by the factor $[1 + (eV/E_c)^2(\Gamma_w^2 - \Phi_i^2)/2(\Gamma_w^2 + \Phi_i^2)^2]$ (for $-\pi < \Phi_i < \pi$ and $\Phi_i \neq 0$). Assuming typical values of the device, $v_d \sim 10^5$ m/s and $L \sim 0.7$ μm, for $V \sim 50$ μV, the possible value of $eV/E_c \approx 0.5$ [cf. Fig. 2(d)]. Further, for $eV/E_c = 2\Gamma_w$, the sensitivity of the detector is reduced up to 80%. The value of Γ_w can be made as large as 1 by changing R_1 and R_2. Thus with the parameters given above, biasing up to ~ 100 μV can be applied for high-sensitivity measurements. The range of high-sensitivity measurements can be increased for smaller devices since E_c is inversely proportional to L.

In order to understand the measurement process and the back action of the detector, we consider evolution of the combined system of detector and qubit. When an electron is injected from source α and the initial state of qubit is $a_0|0\rangle + a_1|1\rangle$, the state of the combined qubit-detector system evolves as

$$|\psi\rangle = (a_0|0\rangle + a_1|1\rangle)c_\alpha^\dagger|F\rangle \rightarrow a_0|0\rangle|\xi_0\rangle + a_1|1\rangle|\xi_1\rangle, \quad (6)$$

where $|F\rangle$ denotes Fermi sea of all the electrodes and $|\xi_i\rangle = (\bar{r}_i c_\beta^\dagger + \bar{t}_i c_\delta^\dagger)|F\rangle$ for $i = 0, 1$ are detector states. The final state of the qubit is given by the reduced density matrix, ρ

$=Tr_{\text{det}}|\psi\rangle\langle\psi|$, obtained after tracing over the detector states. The dephasing of qubit can be expressed in terms of off-diagonal element of density matrix ρ as $|\rho_{01}(t)|$ $=|\rho_{01}(0)|\exp(-\Gamma_d t)$, where Γ_d, detector back-action-induced dephasing rate, is given by $\Gamma_d = -h^{-1}\int d\epsilon \log|\bar{r}_0\bar{r}_1^* + \bar{t}_0\bar{t}_1^*|$.[2,16] In the linear regime, for weak measurement ($|\bar{r}_0\bar{r}_1^* + \bar{t}_0\bar{t}_1^*| \sim 1$), the dephasing rate Γ_d can be expanded in terms of the change in the transmission probability, $\Delta T = |\bar{t}_0|^2 - |\bar{t}_1|^2$ and the change in the relative scattering phase $\Delta\zeta = \arg(\bar{t}_1/\bar{r}_1)$ $-\arg(\bar{t}_0/\bar{r}_0)$ as follows:

$$\Gamma_d = \Gamma_T + \Gamma_\zeta, \qquad (7a)$$

$$\Gamma_T = \frac{eV}{8h}\frac{(\Delta T)^2}{T(1-T)}, \quad \Gamma_\zeta = \frac{eV}{2h}T(1-T)(\Delta\zeta)^2, \qquad (7b)$$

where $T = (|\bar{t}_1|^2 + |\bar{t}_0|^2)/2$. Note that the validity of this relation is limited by $\Gamma_d \ll eV/h$ because of the assumption of continuous weak measurement. The information of the qubit state is reflected in the change in source-drain current. Therefore only the information of the qubit in Γ_T part of dephasing is utilized by the detector. One can find that the measurement rate of the detector Γ_m is equal to Γ_T. However, the information lost in the part of dephasing Γ_ζ goes undetected. For a quantum-limited detector it is necessary that the unutilized information in phases should be eliminated; i.e., $\Delta\zeta = 0$. In a single QPC detector that obeys mirror-reflection symmetry and time-reflection symmetry the relative phase between transmission and reflection amplitude remains constant and change in relative phase vanishes ($\Delta\zeta = 0$).[7,18,19]

From Eqs. (2) and (3) change in relative phases between transmission and reflection amplitudes for Fabry-Perot interferometer is given by

$$\Delta\zeta = \arg\left\{ e^{i\Delta\theta}\frac{\sqrt{R_a} - \sqrt{R_b}e^{i\Phi_0}}{\sqrt{R_a} - \sqrt{R_b}e^{i(\Phi_0+\Delta\theta)}} \right\}. \qquad (8)$$

For $R_a = R_b$, from Eq. (8), we get $\Delta\zeta = \Delta\theta/2 + \pi$, for $0 > \Phi_0 > -\Delta\theta/2$ and $\Delta\zeta = \Delta\theta/2$, otherwise.

Therefore, in the case when both QPCs in Fabry-Perot interferometer have same reflection probabilities ($R_a = R_b$), $\Delta\zeta$ always remains nonzero. This implies that there is always some information loss in the phases which goes undetected and detector cannot perform quantum-limited measurement. Note that this behavior is different from the detection with resonant transmission at zero magnetic field,[20] where the quantum-limited detection is possible only for symmetric double QPCs. In Fig. 2(a) we show measurement rate Γ_m and dephasing rate Γ_d calculated from Eq. (7) for $R_a = R_b$. We find that dephasing rate of the qubit is always higher than the measurement rate. In this case some information is always lost in scattering phases, which means quantum-limited measurement is not possible. However, for higher values of R_a and R_b, $\Gamma_T \gg \Gamma_\zeta$ and the measurement is nearly quantum limited except at resonance. At resonance relative scattering phase $\Delta\zeta$ faces an abrupt change by π which results in maximum loss of information.

From Eq. (8), change in relative scattering phases for $R_a \neq R_b$ is given by

$$\Delta\zeta = \frac{\Delta\theta}{2} + \tan^{-1}\left[\frac{(R_a - R_b)\sin\left(\frac{\Delta\theta}{2}\right)}{(R_a + R_b)\cos\left(\frac{\Delta\theta}{2}\right) - 2\sqrt{R_a R_b}\cos\left(\frac{\Phi_0+\Delta\theta}{2}\right)} \right]. \qquad (9)$$

In this case, we find the condition for quantum-limited measurement $\Delta\zeta = 0$ simplifies to $R_a/R_b = \cos^2(\Phi_0 + \Delta\theta/2)/\cos^2(\Delta\theta/2)$. For small value of $\Delta\theta$, $\cos^2(\Phi_0 + \Delta\theta/2)/\cos^2(\Delta\theta/2)$ is always less than unity except at $\Phi_0 = 0$ where measurement is not possible. This clearly shows that in Fabry-Perot interferometer quantum-limited measurement can only be possible if $R_a < R_b$ and the value of Φ_0 for quantum-limited measurement is given by $\Phi_0 \approx \pm\cos^{-1}\sqrt{R_a/R_b}$. In Figs. 2(b)–2(d), we show dephasing rate and measurement rate of qubit for Fabry-Perot interferometer having QPCs with different reflection probabilities ($R_a \neq R_b$). For $R_a > R_b$, shown in Fig. 2(b), dephasing rate is always larger than the measurement rate. This shows that the detector has poor efficiency for such construction. On the other hand, in Fig. 2(c) for $R_a < R_b$, there exist two points where the measurement rate is equal to the dephasing rate at $\Phi_0 \simeq \pm\cos^{-1}\sqrt{R_a/R_b}$. These points are symmetrically placed on both sides of resonance. For finite bias we average over the energy of the injected electrons. We find that at small bias $eV/E_c = 0.5 \lesssim \Gamma_w$ [see Fig. 2(d)], our results are not modified much. The measurement rate is reduced very much at large biasing, $eV/E_c \gg \Gamma_w$ and the quantum-limited operation of the detector is not possible. Similarly, we also found that (not shown here) thermal broadening at high temperature ($kT/E_c \gg \Gamma_w$) reduces the sensitivity and the efficiency.

If we also include effect of environment on the qubit, the coupling to the environment relaxes the state of the qubit to its lower energy state. The condition when environment can produce dephasing and the measurement of relaxation rate has been discussed in detail in Ref. 21. Coupling of the qubit with environment can reduce the efficiency of the detector only when environment also produces dephasing.

Our findings are unique because of the following facts. For a single QPC as a quantum-limited charge detector, satisfaction of time-reversal symmetry and mirror-reflection symmetry is essential.[7,18,19] Technically construction of such QPC may not be trivial and the information loss is usually large for generic QPC. Typically, the dephasing rate is reported larger than the measurement rate by more than 1 order of magnitude.[3,22,23] Here we report that in Fabry-Perot interferometer quantum-limited measurement is possible if the first QPC has smaller reflection than the second QPC. Further, this Fabry-Perot construction provides much higher precision than a two-path (Mach-Zehnder) interferometer does, due to multiple reflections of electrons.

Next, we briefly discuss possibility of further improvement in sensitivity using quantum entanglement. For our purpose we consider spin-entangled singlet pairs injected through identically biased input terminals α and γ. The state of injected electrons can be expressed as $|\psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(c_{\alpha\uparrow}c_{\gamma\downarrow} - c_{\alpha\downarrow}c_{\gamma\uparrow})|F\rangle$, where $\uparrow$ and $\downarrow$ represent up and down spin of an electron. Methods for production and transport of spin-entangled electrons in solid-state structures have been discussed in Ref. 24. For this input state electrons show bunch-

ing behavior and the shot noise in the interferometer is enhanced.[25] Electron bunching, in turn, leads to improvement in sensitivity. For each up or down-spin Fermi operators in this state scattering matrix is given by Eq. (1). The final state of the two electrons at drains β and δ is given by (for the qubit charge i)

$$|\psi_f^i\rangle = \sqrt{2}\left[\bar{r}_i \bar{t}_i' c_{\beta\uparrow}^\dagger c_{\beta\downarrow}^\dagger + \bar{t}_i \bar{r}_i' c_{\delta\uparrow}^\dagger c_{\delta\downarrow}^\dagger + \frac{1}{2}(\bar{t}_i \bar{t}_i' + \bar{r}_i \bar{r}_i')(c_{\beta\uparrow}^\dagger c_{\delta\downarrow}^\dagger \right.$$
$$\left. + c_{\delta\uparrow}^\dagger c_{\beta\downarrow}^\dagger) \right]|F\rangle. \tag{10}$$

From this state one finds that the dephasing rate of the qubit as[12]

$$\Gamma_d^s = \frac{eV}{h}\frac{(\Delta T)^2}{T(1-T)} + 4\frac{eV}{h}T(1-T)(\Delta\zeta)^2. \tag{11}$$

The dephasing rate Γ_d^s is enhanced by a factor of 8 compared to the case of injecting independent electrons at a single input [Eq. (7)]. Taking into account biasing two inputs with spin degeneracy in Eq. (11), the charge sensitivity (per electron) of the singlet state is enhanced by a factor of 2.[26] The average current at the output β or δ is independent of the phase change $\Delta\phi$. In order to detect the phase shift $\Delta\phi$, it is necessary to measure shot noise or cross correlation at the output leads.

In conclusion, we have discussed high-sensitivity quantum-limited charge detection using electronic Fabry-Perot interferometer with edge states. We note that in the realization of electronic Mach-Zehnder interferometer significance of electron-electron interactions at nonlinear bias[27] and temperature dependence on dephasing[17] have been reported. Such studies in our scheme may also have experimental relevance.

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